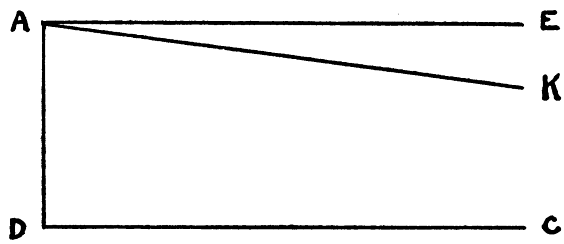
lines that cut DC? Does it make an angle with the parallel or is it the same line?"

Consider the same argument in connection with the following figure in orthodox Euclidean geometry, where AE and DC are perpendicular to AD and are therefore parallel. All other lines (or *rays*) in the angle DAE intersect DC. Let AK be "the last of the lines that cut DC. Does it make an angle with the parallel or is it the same line?....



"If the lines make an angle I suppose that that angle can be bisected, indeed *n*-sected, and such section-lines will be lines that neither cut nor non-cut. If the lines are only one single line then we have a line that both cuts and non-cuts."

Of course this argument is entirely fallacious, but it applies equally badly, nevertheless, to the geometry of Euclid and to Lobatchevsky. Other portions of the article are, of course, equally vulnerable.

G. W. Greenwood, M. A. (Oxon).

Dunbar, Pa.

Mr. F. C. RUSSELL STILL DEMURS.

To the Editor of The Monist:

I wish to thank the Editor for his considerate notice of my article "A Modern Zeno" in *The Monist* of April, 1909. I think, however, that he is in error as to my assumption of the straight line. It was my special and paramount solicitude to avoid that assumption, and it seems to me that I have succeeded. But a discussion of the points involved would make this reply too prolix.

I intended to make, and I thought I made, my article a distinct *plea for better information.* I judged myself an example of a numerous class who seem to themselves to have good geometrical faculty, and who are warranted in that persuasion by a body of confirmations independent of their own esteem, and yet who are per-

plexed and mystified as they study to understand the non-Euclidean doctrines. So I judged it to be eminently conducive to my purpose to exemplify in my article the manner and fashion after which such minds as mine are apt to conceive and deal with the elements of geometry. I hoped that my gropings would more or less reveal to the non-Euclideans the matter or matters at fault in my class of minds, and that some one or more of them would take the pains to so explain their doctrine as to put it within our compass.

I am a little surprised to observe that some of my critics presume a hierarchy in the domain of mathematics and would have the truths of geometry and the issues arising therein depend upon the authority of that hierarchy. Now while I am in no wise indisposed to defer largely to such an authority, I must protest that any blind subjection would outrage the crowning honor of mathematics, viz., that, unique among the sciences in that regard, it asks absolutely nothing on the ground of authority but appeals solely to insight and reason. Geometry, especially, walks by sight and not by faith.

Besides, the matters I agitate pertain to the very elements of geometry, and as to these how is it that the professional expert has, on account merely of his professional expertness, so much the advantage of the amateur? Of course professional expertness is an index of intellectual quality, but if other things be equal (an important condition truly) how is the professional expert better fitted to see more lucidly in dealing with the elements of geometry than any other person of good geometric faculty?

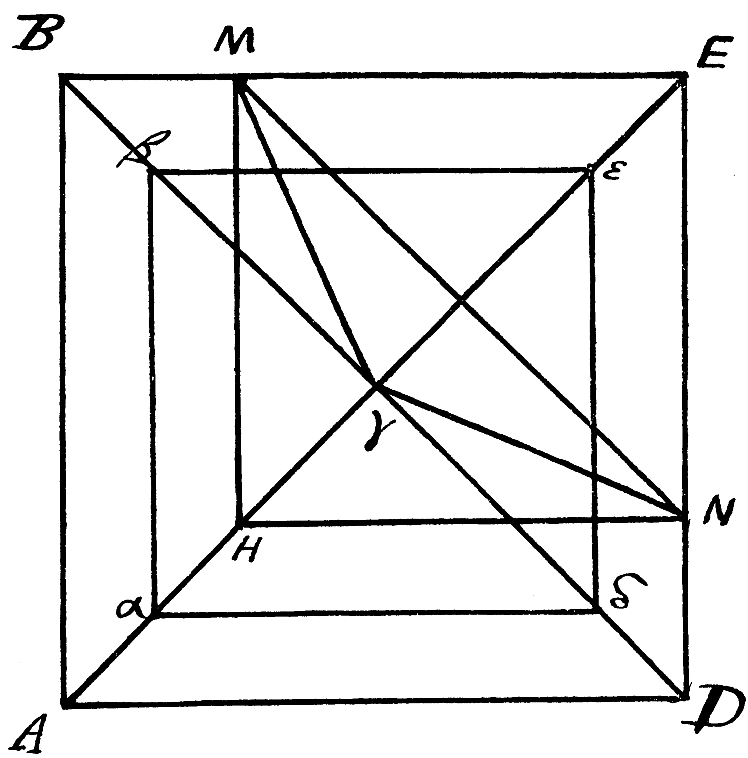
Since all of my professional critics have gone at once at that discourse of mine concerning the right-angled isosceles triangle I take it that my doctrine in that point is regarded as conspicuously vulnerable. I said in my article, "The proof that the two secondary triangles are exactly equal to one another, that they are right-angled and isosceles, and that the four tertiary triangles are in all respects precisely in the same case, is so simple in more than one way that it would be almost an imputation upon the reader to spread it before him." In saying this I was guilty of a mortifying inadvertance and of an unwarrantable presumption. Still, unless I am very, very sadly mistaken, the doctrine I laid down is quite sound and can be geometrically proved. So as a further exemplification of the geometrical inveteracies of such minds as mine I will now spread before the reader in detail what seems to me to be good geometrical proof of my proposition.

Consider and refer to the following figure.

Here are three quadrilateral figures ABED, $\alpha\beta\epsilon\delta$ and HMEN. They are really squares, but as yet we do not know that and so we will for the present call them *even rhombs.* (The word "even" will get its justification in due course.) ABED we will call the *outer* even rhombus, $\alpha\beta\epsilon\delta$ the *inner* even rhombus and HMEN the *corner* even rhombus. $\alpha\beta\epsilon\delta$ and HMEN are equal as we shall see. Each rhombus has two sets of triangles, for example, in the outer even rhombus ABED such triangles as ABE, BED, etc., to be called here its *major* triangles, and such triangles as AγB, BγE, etc., to be called here its *minor* triangles. So far all is loose preliminary, intended only as an aid in understanding the language I use.

The figure is constructed as follows:

Draw the straight line AαH$\gamma\epsilon$E and the straight line B$\beta\gamma\delta$D so that they intersect one another at γ at right angles. Take the



points α, β, ϵ and δ so that any one of the intervals $\gamma\alpha$, $\gamma\beta$, $\gamma\epsilon$ and $\gamma\delta$ shall be equal to any other of them. Join α and β, α and δ, ϵ and β, and ϵ and δ, by right lines. The four minor triangles $\alpha\gamma\beta$, $\alpha\gamma\delta$, $\epsilon\gamma\beta$

and $\epsilon\gamma\delta$ are made. Since any and every one of these triangles have been made right-angled and equal-sided about the right angle and any one side equal with any other, any one of the triangles is equal to any other of them, and hence any one of the sides $\alpha\beta$, $\alpha\delta$, $\epsilon\beta$ and $\epsilon\delta$ is equal to any other of them. Furthermore on account of the equality and isosceles nature of these (minor) triangles any one of the eight angles $\alpha\beta\gamma$, $\beta\epsilon\gamma$, $\epsilon\delta\gamma$, $\delta\alpha\gamma$, $\alpha\delta\gamma$, $\delta\epsilon\gamma$, $\epsilon\beta\gamma$ and $\beta\alpha\gamma$ is equal to any other of them. Since we do not as yet know how these angles last mentioned compare with the right angle, and since it will be necessary to have immediately a name for them we will for the present call such angles *u-angles.* These *u*-angles are not in any wise indeterminate. They are just as determinate as is the right angle, and they might be defined as being such angles as the sides of an isosceles right-angled triangle make with the hypotenuse. Only it is not yet determined how they compare with the right angle.

The angles $\alpha\beta\epsilon$, $\alpha\delta\epsilon$, $\beta\epsilon\delta$ and $\beta\alpha\delta$ being each and every one of them composed of two *u-angles,* are, on that account, as yet undetermined in their relations to the right angle, but they are indeterminate in no other respect. We will for the present call such angles *w-angles.* Any one of them is equal to any other of them. The four *major* triangles of the inner even rhombus, $\alpha\beta\delta$, $\epsilon\beta\delta$, $\alpha\beta\epsilon$ and $\alpha\delta\epsilon$, being each *w*-angles between pairs of equal sides, any one of which sides is equal to any other of them, are any one of such triangles equal to any other of them. The thoroughgoing evenness of the inner rhombus should now, I think, be abundantly manifest.

Now take A and E on the line AαH$\gamma\epsilon$E and B and D on the line B$\beta\gamma\delta$D so that any one of the intervals γA, γB, γE and γD shall be equal to either one of the (equal) sides of the inner even rhombus. Join A and B, A and D, E and B and E and D with right lines. Then there will be made an outer even rhombus with minor triangles, the sides of the rhombus, the angles of the minor triangles, the corner angles of the rhombus, the major triangles, etc., all equal homologously as in the inner even rhombus, such angles as ABD, BAE, etc., being *u*-angles and such angles as ABE, BED, etc., being *w*-angles.

Now take on BE the point M so that the interval EM shall equal the interval Eγ (or the equal interval $\beta\epsilon$, or etc.) and take on ED the point N so that the interval EN shall equal the same interval as above prescribed for the interval EM. Take on the line AαH$\gamma\epsilon$E the point H so that the interval EH shall equal the interval BE

(or the equal interval AB or etc.). Join M and N, M and H, M and γ, N and γ, and N and H with straight lines.

Now pursuant to Euclid I-V, MH equals Bγ which equals Eγ, either being equal to $\beta\epsilon$ (or etc.) which equals ME which equals EN (or etc.), and pursuant to the same Euclidean theorem, NH equals Dγ which equals Bγ, etc., so that MH equals NH and so that any one of the four sides, MH, NH, ME and NE, is equal to any one of the others. Now the angle MEN being a w-angle equals the angle $\beta\epsilon\delta$, while the sides of the triangle MEN, viz., ME and NE, are both equal to each other and either side equal to either side of the triangle $\beta\epsilon\delta$. Hence the two triangles MEN and $\beta\epsilon\delta$ are equal. But the triangle NHM equals the triangle MEN (three-sides equal). Now the angles MHN and MEN have heen shown to be both w-angles, and since the triangle MEN is equal to the triangle $\beta\epsilon\delta$ it is further shown that the angle EMN which corresponds to the angle $\epsilon\beta\delta$ (or etc.) is a u-angle and that the angle ENM is also a u-angle is shown by a precisely like argument. But since the triangles EMN and HMN are equal, the angles HMN and HNM are also u-angles, so that the angles HME and HNE are shown to be w-angles. Now the triangles $\alpha\beta\epsilon$ and HME have the angles $\alpha\beta\epsilon$ and HME equal to one another (both being w-angles), which angles are in either triangle included between a pair of sides equal to each other and equal any one such side in either triangle to any such side in the other triangle. Hence the two triangles are equal and the side $\alpha\epsilon$ is equal to the side HE which was made equal to the side BE of the outer even rhombus. But it has just been shown that in the triangle $\alpha\beta\epsilon$ the side $\alpha\epsilon$ is equal to the side BE of the triangle BγE, and it was heretofore shown that the side Bγ (or Eγ) was equal to the side $\beta\epsilon$ (or $\beta\alpha$). Hence the triangle $\alpha\beta\epsilon$ is equal to the triangle BγE and the angle $\alpha\beta\epsilon$ homologous to the angle BγE is equal to the same. But BγE is a right angle. Hence the until now named w-angle $\alpha\beta\epsilon$ is now shown to be no other than a right angle, and its half, the until now called u-angle, is shown to be precisely half of a right angle.

The rest now goes almost of itself. In a right-angled isosceles triangle the acute angles are half right angles and equal to either one of the sections of the bisected right angle of the triangle. Hence in such a right-angled isosceles triangle the line from the vertex of the right angle to the mid point of the hypotenuse divides the primary triangle into two equal *isosceles* right-angled triangles, and the bisecting line is precisely one-half of the hypotenuse. Of course if

the above argument is sound the angle-sum of the right-angled isosceles triangles, at least, is precisely two right angles. If this is true I suppose it to be not very difficult to prove first that the angle-sum of *any* right-angled triangle is the same, and then that the angle-sum of any triangle is the same. There is very possibly some flaw in my course of argument. I can only say that up to the present time I have not been able to detect it.

It is objected against my remarks on the system of Lobatchevsky beginning about the middle of page 301, Vol. XIX of *The Monist* (April, 1909 number) that I have ignored the fact that Lobatchevsky distinguishes between sides in parallelism and that the statement of his Theorem 25 ought to be glossed by inserting the words "on the same side" in about the middle of that statement. Some of my critics make this gloss in their statement of said theorem. I avow that I honestly thought that the omission of the condition was deliberately designed by Lobatchevsky, for it seems to me that the reason of the matter justified the omission. Let us see. Lobatchevsky says in effect (Theorem 16—[*Monist,* Vol. XIX, pp. 291-292]) that in the uncertainty that obtains whether there may not be other lines than the perpendicular AE that do not cut DC, he will assume that such lines are possible, in plurality. The boundary line of such lines he takes as his parallel and, of course, makes it make the angle $\Pi(p)$ an angle less than a right angle. This leads him to remark that on the assumption he makes there will be two lines through the same point both parallel to the BDC line. This is *his* distinction of *sides in parallelism,* and it goes no further. As to such an idea as that two lines may be parallel if they are taken in the same *sense,* and yet *not* parallel if taken in opposite senses, I fail to find any vestige of it in Lobatchevsky's text. That would be to make Lobatchevsky's system a system of vectors instead of a geometry, and I am sure such a system as well as the idea of a *sensed relation* would put me to permanent intellectual confusion, should I endeavor to find any sense in either of them.

But Lobatchevsky, in his Theorem 17, stated thus, *"A straight line maintains the characteristic of parallelism at all its points,"* shows by his figure and demonstration that he had plainly in mind that a parallel was parallel as well on the other side of the Π (p) line as on the one side. So I fail to see how my figure on page 301, April 1909, *Monist,* and my remarks in connection therewith ignore or violate any of the principles laid down by Lobatchevsky. I did not aver that he drew the consequences that I did. I plainly started

out with the remark, "But it is time to search for results ourselves," and it seems to me that I showed that the principles of Lobatchevsky lead to contradiction. At any rate, I cannot see how the distinction of *sides in parallelism* avoids the consequences I drew. It is true enough that Lobatchevsky, keeping (with the single exception I have mentioned) always on one side of the $\Pi(p)$ line, falls into no contradiction. With a parallel differing only infinitesimally from the ordinary parallel, and keeping always on the same side of the $\Pi(p)$ line, how could he fall into contradiction? It may be noticed in passing that Lobatchevsky makes nothing whatever turn upon any of the assumed plurality of lines that lie between his parallel and the perpendicular AE. It is probably of no consequence unless for the notice it gives us that so far as the system of Lobatchevsky is concerned there are no lines on the same side that pass through A that are of any consequence except the perpendicular AE and the parallel AH, the latter differing from AE by only an infinitesimal shade.

I cannot admit that my definitions of the straight line and the plane are amplifications of definitions previously published in Professor Halsted's *Rational Geometry.* Had I so esteemed them or either of them, I should not have published them as my own. It would be quite idle, however, for us two to dispute over the matter. We are both on record and whoever feels interest enough in the issue to inquire will decide irrespective of any clamor of ours. I may say, however, that definitions are a matter of words, apt for the publication of enough of the proper marks of the thing defined to make fully determinate all the other proper marks without making any use of the thing defined either expressly or by implication. I do not, as does Professor Halsted, make the straight line and the plane "aggregates of points." True, in leading up to my definition I make use of triads and aggregates of points. But when all is ready I drop those ideas and define a straight line as a certain kind of a *line,* and a plane as a certain kind of a *surface,* neither of which would I think of defining as an aggregate of points. As a matter of fact much of my method in geometry is the result of a practical business with linkages. Almost any one can see that my straight range is a virtual though mechanically unrealizable linkage. I may say that I have a linkage of thirty-seven links, any link of which is identical with any other link, with which linkage with two points fixed I can by continuous motion draw a limited straight line (in fact two) in line with the two fixed points. But this linkage did not

reveal the essence of the straightness of the straight line, as did the straight range. The latter is three-dimensional. The former only planar.

If it be asked what use can be made of my definition of the straight line, I can only say that I have not as yet found it of as much use in elementary geometry as I had anticipated. I can say, however, that it follows quite readily from the definition that two-intersecting straight lines can have only the intersecting point in common, and quite as readily that the straight line cannot return into itself; that is to say, that the straight line is infinite, a result which alone would sadly mar the symmetry of the non-Euclidean system.

CHICAGO, ILL. FRANCIS C. RUSSELL.

AN OPEN LETTER.

To the Editor of The Monist:

It was with great interest that I read your reply, in *The Monist* for July, to my article entitled "A Biochemical Conception of the Phenomena of Memory and Sensation" which appeared in the same number. It is not my intention to attempt anything approaching an exhaustive critique of the philosophical position which you assume, —an attempt which, as some four thousand years of sterile discussion have demonstrated, would be entirely useless. I am nevertheless constrained to draw your attention to certain points in which I am of the opinion that you have not represented my position, and that of a number of scientific colleagues, with that fairness which, I believe, we have a right to expect from the editor of a journal "Devoted to the Philosophy of Science." Since a charge of misrepresentation affects not only the accused, but also his readers, I have taken the liberty, Sir, of addressing you in the time-honored form of an "open letter."

In the first place, Sir, I must take exception to the style in which you have expressed yourself concerning my formulation of my hypothesis of memory in mathematical terms, and in which you have alluded to Professor Loeb's term "associative hysteresis," which he has proposed to substitute, in scientific literature, for the popular term "memory." I know that any suggestion that the mathematical or scientific author is seeking to "impress" or mystify his readers by the use of mathematical symbols or of scientific terms is welcomed by that type of general reader, who, with the common dislike of